CHAPTER (5)

CONTROL VOLUME APPROACH

SOLVED PROBLEMS

CONTROL VOLUME APPROACH

	Volume Flow Rate	Mass Flow Rate
1. Velocity is constant:	$\dot{Q} = AV$	$\dot{m} = \rho\dot{Q} = \rho AV$
2. Velocity is variable:	$\dot{Q} = \int_A V dA$	$\dot{m} = \int_A \rho V dA = \rho \int_A V dA = \rho\dot{Q}$

3. Continuity equation $\frac{dM_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out}$

4. Continuity equation in a pipe $\dot{m}_1 = \dot{m}_2$ $(\rho AV)_1 = (\rho AV)_2$

5. The extensive property is defined as (B)

6. The intensive property $\left(b = \frac{B}{mass} \right)$

CONTROL VOLUME APPROACH

7. **Reynolds transport theorem(in general)**

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} b\rho dQ + \int_{cs} b\rho V \bullet dA$$

8. **Reynolds transport theorem(for mass)**

$$\frac{dM_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho dV + \int_{CS} \rho V \bullet dA$$

9. **Cavitation index**

$$\sigma = \frac{p_0 - p_V}{\frac{1}{2}\rho V_1^{\,2}}$$

10. **Differential form of continuity equation**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

CONTROL VOLUME APPROACH

PROBLEM 5.2

Situation: Water flows in a 16 in pipe. $V = 3$ ft/s.

Find: Discharge in cfs and gpm.

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= (3 \text{ ft/s})(\pi/4 \times 1.333^2) \\ & \boxed{Q = 4.19 \text{ ft}^3/\text{s}} \\ &= (4.17 \text{ ft}^3/\text{s})(449 \text{ gpm/ft}^3/\text{s}) \\ & \boxed{Q = 1880 \text{ gpm}} \end{aligned}$$

CONTROL VOLUME APPROACH

PROBLEM 5.4

Situation: An 8 cm. pipe carries air, $V = 20$ m/s, $T = 20^oC$, $p = 200$ kPa-abs.

Find: Mass flow rate: $\dot{m}$

ANALYSIS

Ideal gas law

$$\begin{aligned} \rho &= p/RT \\ &= 200,000/(287 \times 293) \\ \rho &= 2.378\ kg/m^3 \end{aligned}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho V A \\ &= 2.378 \times 20 \times (\pi \times 0.08^2/4) \end{aligned}$$

$$\boxed{\dot{m} = 0.239 \text{ kg/s}}$$

PROBLEM 5.10

Situation: Water flows in a 4 ft pipe. The velocity profile is linear. The center lin
velocity is $V_{max} = 15$ ft/s. The velocity at the wall is $V_{min} = 12$ ft/s.

Find: Discharge in cfs and gpm.

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$
\begin{aligned}
Q &= \int_A V dA \\
&= \int_0^{r_0} V 2\pi r dr
\end{aligned}
$$

where $V = V_{max} - 3r/r_0$.

$$
\begin{aligned}
Q &= \int_0^{r_0} (V_{max} - (3r/r_0))2\pi r dr \\
&= 2\pi r_0^2((V_{max}/2) - (3/3)) \\
&= 2\pi \times 4.00((15/2) - (3/3)) \\
& \boxed{Q = 163.4 \text{ cfs}} \\
&= 163.4 \times 449 \\
& \boxed{Q = 73,370 \text{ gpm}}
\end{aligned}
$$

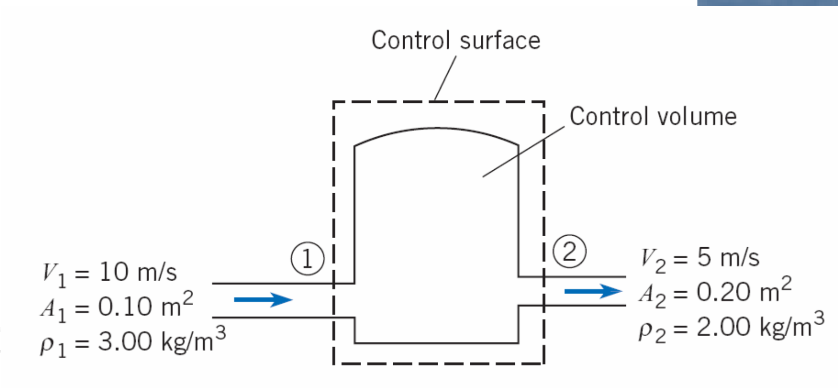
PROBLEM 5.34

Situation: Mass is flowing into and out of a tank

Find: Select the statement that is true.

ANALYSIS

Mass flow out



$$\begin{aligned} \dot{m}_o &= (\rho A V)_2 \\ &= 2 \times 0.2 \\ &= 2 \text{ kg/s} \end{aligned}$$

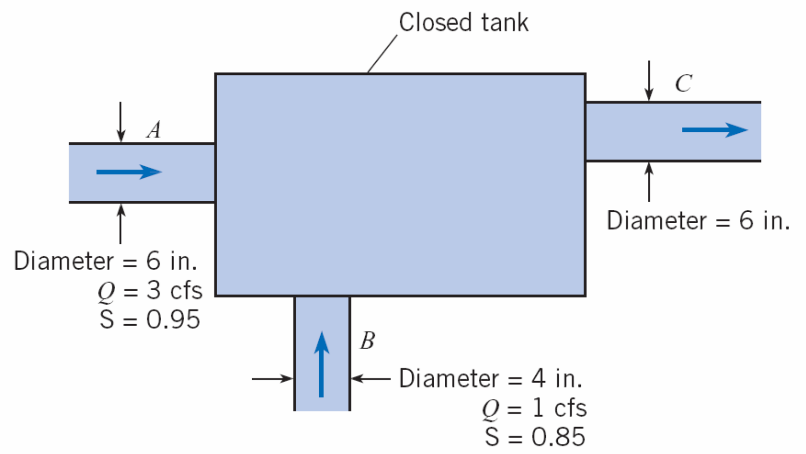
Mass flow in

$$\begin{aligned} \dot{m}_i &= (\rho A V)_1 \\ &= 3 \times 0.1 \times 10 \\ &= 3 \text{ kg/s} \end{aligned}$$

Only selection (b) is valid.

PROBLEM 5.54

Situation: Flows with different specific weights enter a closed tank through ports A and B and exit the tank through port C. Assume steady flow. Details are provided on figure with problem statement.



Find: At section C:
(a) Mass flow rate.
(b) Average velocity.
(c) Specific gravity of the mixture.

Assumptions: Steady state.

APPROACH

Apply the continuity principle and the flow rate equation.

ANALYSIS

Continuity principle

$$\begin{aligned} \sum \dot{m}_i - \sum \dot{m}_o &= 0 \\ -\rho_A V_A A_A - \rho_B V_B A_B + \rho_C V_C A_C &= 0 \\ \rho_C V_C A_C &= 0.95 \times 1.94 \times 3 + 0.85 \times 1.94 \times 1 \\ & \boxed{\dot{m} = 7.18 \text{ slugs/s}} \end{aligned}$$

Continuity principle, assuming incompressible flow

$$\begin{aligned} V_C A_C &= V_A A_A + V_B A_B \\ &= 3 + 1 = 4 \text{ cfs} \end{aligned}$$

Flow rate equation

$$\begin{aligned} V_C &= Q/A = 4/[\pi/4(1/2)^2] \\ & \boxed{=20.4 \text{ ft/s}} \\ \rho_C &= 7.18/4 = 1.795 \text{ slugs/ft}^3 \\ S &= 1.795/1.94 \\ & \boxed{S = 0.925} \end{aligned}$$

PROBLEM 5.55

Situation: O_2 and CH_4 enter a mixer, each with a velocity of 5 m/s. Mixer conditions: 200 kPa-abs., 100 °C. Outlet density: $\rho = 2.2\ \text{kg/m}^3$. Flow areas: 1 cm^2 for the CH_4, 3 cm^2 for the O_2, and 3 cm^2 for the exit mixture.

Find: Exit velocity of the gas mixture: V_{exit}

APPROACH

Apply the ideal gas law to find inlet density. Then apply the continuity principle.

ANALYSIS

Ideal gas law

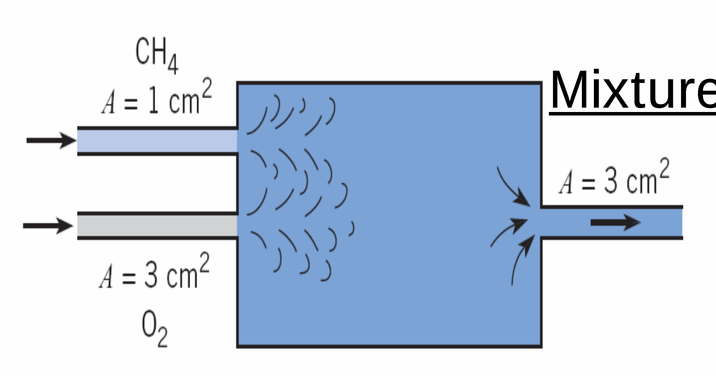
$$\begin{aligned}\rho_{O_2} &= p/RT \\ &= 200,000/(260 \times 373) \\ &= 2.06\ \text{kg/m}^3 \\ \rho_{CH_4} &= 200,000/(518 \times 373) \\ &= 1.03\ \text{kg/m}^3\end{aligned}$$

Continuity principle

$$\begin{aligned}\sum \dot{m}_i &= \sum \dot{m}_o \\ \rho_e V_e A_e &= \rho_{O_2} V_{O_2} A_{O_2} + \rho_{CH_4} V_{CH_4} A_{CH_4} \\ V_e &= (2.06 \times 5 \times 3 + 1.03 \times 5 \times 1)/(2.2 \times 3)\end{aligned}$$

$$\boxed{V_e = 5.46\ \text{m/s}}$$

Problem 5.55 (p. 176)



PROBLEM 5.75

Situation: Cavitation in a venturi section with inlet diameter of 40 cm and throat diameter of 10 cm. Upstream pressure is 120 kPa gage and atmospheric pressure is 100 kPa. Water temperature is 10°C.

Find: Discharge for incipient cavitation.

APPROACH

Apply the continuity principle and the Bernoulli equation.

ANALYSIS

Cavitation will occur when the pressure reaches the vapor pressure of the liquid ($p_V = 1,230$ Pa abs). **Table (5)**

Bernoulli equation

$$p_A + \rho V_A^2/2 = p_{\text{throat}} + \rho V_{\text{throat}}^2/2$$

where $V_A = Q/A_A = Q/((\pi/4) \times 0.40^2)$

Continuity principle

$$V_{\text{throat}} = Q/A_{\text{throat}} = Q/((\pi/4) \times 0.10^2)$$

$$\rho/2(V_{\text{throat}}^2 - V_A^2) = p_A - p_{\text{throat}}$$

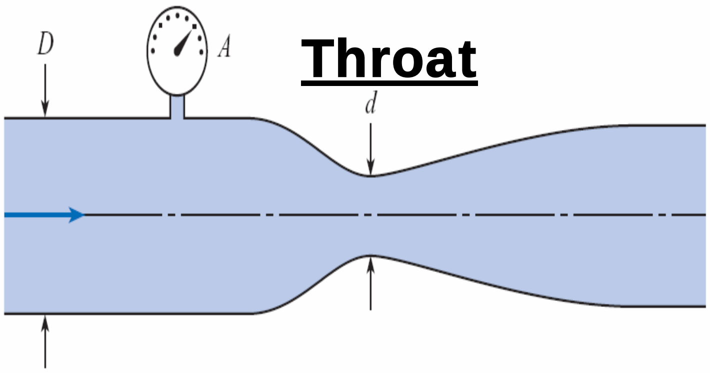
$$(\rho Q^2/2)[1/((\pi/4) \times 0.10^2)^2 - 1/[((\pi/4) \times 0.40^2)^2]$$

$$= 220,000 - 1,230$$

$$500Q^2(16,211 - 63) = 218,770$$

$$\boxed{Q = 0.165 \text{ m}^3/\text{s}}$$

Problem 5.75 (p. 181)



PROBLEM 5.76

Situation: Air with density 0.0644 lbm/ft^3 flows upward in a vertical venturi with area ratio of 0.5. Inlet velocity is 100 ft/s. Two pressure taps connected to manometer with fluid specific weight of 120 lbf/ft^3.

Find: Deflection of manometer.

Assumptions: Uniform air density.

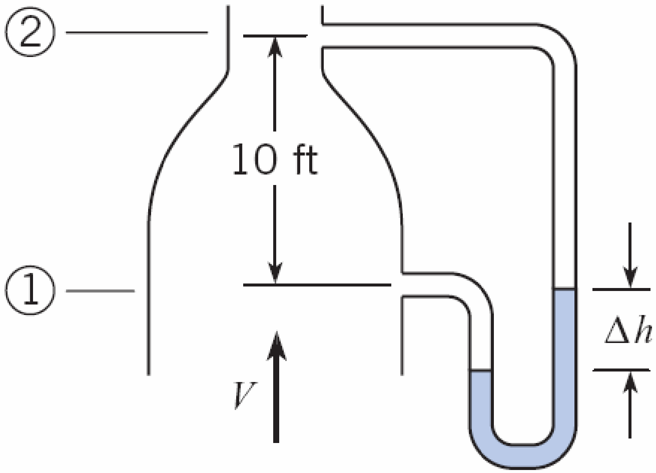
APPROACH

Apply the Bernoulli equation from 1 to 2 and then the continuity principle. Let section 1 be in the large duct where the manometer pipe is connected and section 2 in the smaller duct at the level where the upper manometer pipe is connected.

ANALYSIS

Continuity principle

$$\begin{aligned} V_1 A_1 &= V_2 A_2 \\ V_2 &= V_1 (A_1/A_2) \\ &= 100(2) \\ &= 200 \text{ ft/s} \end{aligned}$$



Bernoulli equation

$$\begin{aligned} p_{z1} + \rho V_1^2/2 &= p_{z2} + \rho V_2^2/2 \\ p_{z1} - p_{z2} &= (1/2)\rho(V_2^2 - V_1^2) \\ &= (1/2)(0.0644/32.2)(40,000 - 10,000) \\ &= 30 \text{ psf} \end{aligned}$$

Manometer equation

$$\begin{aligned} p_{z1} - p_{z2} &= \Delta h(\gamma_{\text{liquid}} - \gamma_{\text{air}}) \\ 30 &= \Delta h(120 - .0644) \\ &\boxed{\Delta h = 0.25 \text{ ft.}} \end{aligned}$$

CONTROL VOLUME APPROACH

THE END